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Modeling of Stress-Strain Behavior of Clay Soil

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Abstract

This paper presents a hyperbolic model suggested by Duncan and Chang to predict the complete stress-strain behavior of clay soil. The hyperbolic model is an incremental non-linear stress-dependent model. The parameters of this model for specific soils can be easily obtained directly from the conventional triaxial tests. In this study to evaluate the model parameters a series of triaxial compression tests were conducted on undisturbed samples collected from the North part of Addis Ababa. The comparison of the simulations with experimental results show that the model is sufficient to reproduce with a very good accuracy the stress-strain behavior of the tested clay soil.

Keywords: Modeling, clay, stress, strain, CD, CU

1. Introduction

For a long time, soil mechanics has been based on Hook's law of linear elasticity for stress and deformation analysis for a soil mass. The stress-strain behavior of soils is not linearly elastic for the entire range of loading of practical interest. In fact, actual behavior of soils is much complicated and they show a great variety of behavior when subjected to different conditions. Idealizations are therefore essential in order to develop simple mathematical model for practical applications. No mathematical model can completely describe the complex behavior of real soils under all conditions. Each soil model aims at a certain class of phenomena, captures their essential features, and disregards what is considered to be of minor importance in that class of applications. As mentioned above, Hook's law has been used successfully in soil mechanics to describe the general behavior of soil media under short term loading condition. But it fails to predict the behavior and strength of a soil-structure interaction problem near ultimate strength condition, because plastic deformation at this load level attains a dominating influence, while elastic deformation becomes of minor importance.

Mathematical modeling of stress-strain curves of soil behavior obtained from triaxial tests gained great interest during the last three decades due to the need for calibration of the recent constitutive soil models that are used in the analysis and prediction of the behavior of complex soil structures and soil/structure interaction problems using finite elements or finite difference method[1]. An overview of constitutive models for soils was given by Lade[6]. The Hyperbolic mathematical model is one of the simpler models that can simulate the nonlinear stress-strain curve of soil. Kondner[3] proposed a functional form based on hyperbolic stress-strain function which developed later by Duncan and Chang[4] .

In this paper a Hyperbolic soil model was employed for simulating the stress-strain curves of clay soil. In order to determine the required material parameters for the hyperbolic model, a series of triaxial compression test were carried out on samples collected from the Northern part of Addis Ababa.

DESCRIPTION OF THE HYPERBOLIC MODEL AND EVALUATION OF MODEL PARAMETERS

DESCRIPTION OF THE MODEL

The hyperbolic model is a variable-parameter model used to simulate the non-linear stress-strain response of soils, and is often used with numerical solution techniques such as the finite element method[2] [9]. The use of hyperbolae was proposed by Kondner[5] to represent the stress-strain behavior of soils. Since the stress-strain behavior of soil depends on confining stress, Duncan and Chang[4] incorporated that effect and used the hyperbola in conjunction with the relationship between the initial modulus and confining pressure proposed by Janbu[7].

A brief summary of the hyperbolic model for soil given by Duncan and Chang[4] and Duncan et al[3] is presented here under. The stress-strain relationship from a constant confining pressure triaxial compression test can be approximated reasonably accurate by a hyperbola (Fig. 1) having the form:

$$(\sigma_1 - \sigma_3) = \frac{\varepsilon}{\frac{1}{E_i} + \frac{\varepsilon}{(\sigma_1 - \sigma_3)_{ult}}} \quad (1)$$

where ε is the axial strain, E_i is the initial tangent modulus, $(\sigma_1 - \sigma_3)_{ult}$ is the ultimate deviator stress at large strain, and $\sigma_1 \text{and} \sigma_3$ are the major and minor principal stresses respectively.

The ultimate deviator stress can be related to the actual failure deviator stress $(\sigma_1-\sigma_3)_f$ by the following equation:

$$R_f = \frac{(\sigma_1-\sigma_3)_f}{(\sigma_1-\sigma_3)_{ult}} \quad (2)$$

where R_f is the failure ratio.

The parameters E_i and $(\sigma_1 - \sigma_3)_{ult}$ for a given stress-strain curve can be determined if the data are plotted on transformed axes, as shown in Fig. 2. The appropriate straight line used to represent the transformed Eq. (1) is

$$\frac{\varepsilon}{(\sigma_1-\sigma_3)} = \frac{1}{E_i} + \frac{\varepsilon}{(\sigma_1-\sigma_3)_{ult}} = a + b\varepsilon \quad (3)$$

In which a and b are the intercept and the slope of the resulting straight line respectively.

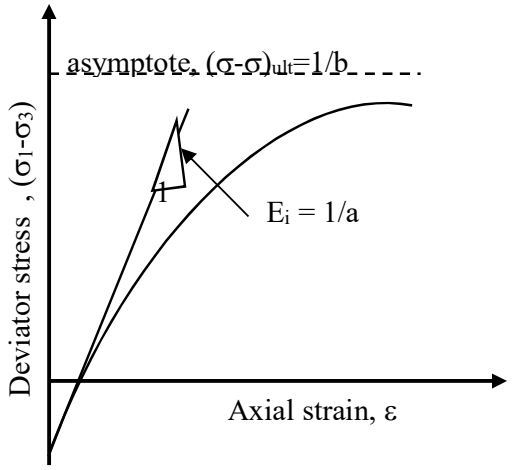


Fig 1. Hyperbolic stress-strain curve[5]

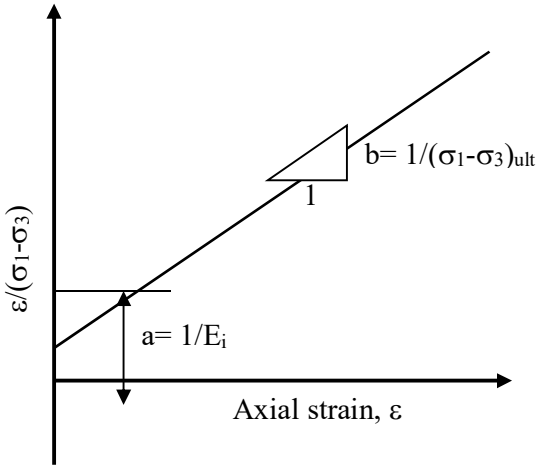


Fig. 2 Transformed hyperbolic stress-strain curve[5]

Experimental studies by Janbu[7] have shown that the relationship between initial tangent modulus and confining pressure may be expressed as:

$$E_i = KP_a\left(\frac{\sigma_3}{P_a}\right)^n \quad (4)$$

where E_i is the initial tangent modulus, σ_3 is the confining pressure, Pa is the atmospheric pressure expressed in the same pressure units as E_i and σ_3, K is a modulus number and n is the exponent determining the rate of variation of E_i with σ_3. The parameters K and n can be determined from a logarithmic plot of E_i against σ_3.

The variation of $(\sigma_1-\sigma_3)_f$ with σ_3 may be expressed conveniently in terms of the Mohr- Coulomb failure criteria as

$$(\sigma_1 - \sigma_3)_f = \frac{2c\cos\phi + 2\sigma_3\sin\phi}{1-\sin\phi} \quad (5)$$

where c is cohesion of the soil, and ϕ is the angle of internal friction.

The tangent modulus, E_t, for any stress state may be determined by differentiating equation (1) with respect to ε and substituting the expressions of equations (2), (4) and (5) into the resulting expression for E_t. The final equation for the tangent modulus is:

$$E_t = \left[1 - \frac{R_f(1-\sin\phi)(\sigma_1-\sigma_3)}{2c\cos\phi + 2\sigma_3\sin\phi}\right]^2 KP_a\left(\frac{\sigma_3}{P_a}\right)^n \quad (6)$$

This equation can be used to calculate the value of E_t for any stress condition if the hyperbolic parameters K, n and R_f and the shear strength parameters, c and ϕ, are known.

EVALUATION OF MODEL PARAMETER

The parameters that needed to be determined in order to execute the hyperbolic model are: K, n, R_f, c and ϕ. These parameters have been determined based on the results of triaxial compression tests conducted on a clay soil samples collected from North of Addis Ababa. The basic properties of the soil are presented in Table 1.

Table 1 Summary of basic soil properties[8]

Location	Semen-Gebeya Area
Clay content (%)	53-68
Liquid limit (%)	57-76
Plastic Limit (%)	24-31
Plasticity index (%	33-47
Specific Gravity	2.70-2.77
Free swell (%)	15-50

In order to evaluate the parameters representing the behavior of the red clay soil upon loading, a series of conventional consolidated undrained and consolidated drained triaxial compression tests were carried out on undisturbed soil samples of the clay soil under investigation[8]. The test results are presented in Tables 2 and 3.

Table 2 Summary of consolidated undrained triaxial compression test results[8]

Confining pressure,σ_3, (kPa)	250	350	450
Deviator stress at failure, $(\sigma_1-\sigma_3)_f$, (kPa)	291.67	399.11	489.02
Pore water pressure at failure, u_w, kPa)	28	42	63
Cohesion, C_{cu}, (kPa)	20		
Angle of internal friction, ϕ_{cu} (deg)	19		

Table 3 Summary of consolidated drained triaxial compression test results[8]

Confining pressure,σ_3, (kPa)	250	350	450
Deviator stress at failure, $(\sigma'_1-\sigma'_3)_f$, (kPa)	313.10	418.47	529.20
Cohesion, C_d (kPa)	15		
Angle of internal friction, ϕ_d (deg)	21.5		

i. **The parameter R_f** :- The stress-strain tests data for the specimens have been plotted on transformed axis (strain/stress vs. strain) as shown in Figs.3 and 4 . Each line in Figs.3 and 4 belongs to a specific confining pressure. The reciprocal of the slope of each line gives $(\sigma_1-\sigma_3)_{ult}$ and the reciprocal of the intercept gives E_i. Then the value of R_f is calculated using Eq. (2). The values of these parameters are presented in Tables 4 and 5.

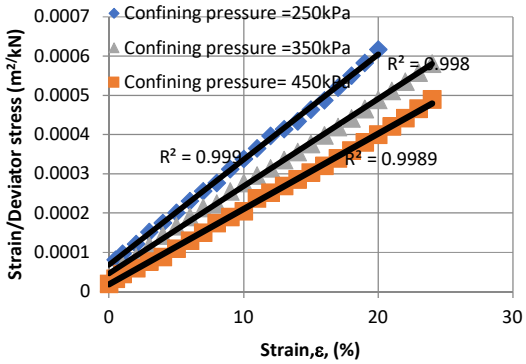


Fig. 3 Transformed stress-strain curves for consolidated undrained triaxial test results

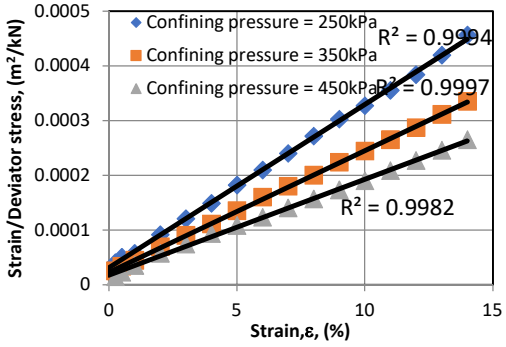


Fig. 4 Transformed stress-strain curves for consolidated drained triaxial test results

Table 4 Determination of parameter R_f from consolidated undrained triaxial tests.

Confining pressure,σ_3,(kPa)	250	350	450
Deviator stress at failure,$(\sigma'_1-\sigma'_3)_f$,(KPa)	291.67	399.11	489.02
Slope (10^{-3})	3.02	2.20	1.77
Intercept($x10^{-5}$)	3.5	2.7	2.2
Initial tangent modulus, E_i=1/intercept, (kPa)	28571.43	37037.04	45454.55
R_f =$(\sigma_1-\sigma_3)_f$ x slope	0.881	0.880	0.864

Table 5. Determination of parameter R_f from consolidated drained triaxial tests

Confining pressure,σ_3, (kPa)	250	350	450
Deviator stress at failure,$(\sigma'_1-\sigma'_3)_f$, KPa	313.10	418.47	529.20
Slope (10^{-3})	3.1	2.32	1.83
Intercept($x10^{-5}$)	3	2.3	1.8
Initial tangent modulus, E_i=1/intercept, (kPa)	33333.33	43478.26	55555.56
$R_f = (\sigma_1-\sigma_3)_f$ x slope	0.964	0.969	0.971

ii. **The parameters K and n**:- The values of initial tangent modulus, E_i, determined from the tests and tabulated in Tables 4 and 5 have been plotted against the corresponding values of confining pressures in a log-log plot, for the purpose of determining the values of K and n. Equation (4) can be written as:

$$\log_{10}\left(\frac{E_i}{P_a}\right) = \log_{10}(K) + n\log_{10}\left(\frac{\sigma_3}{P_a}\right) \qquad (7)$$

Eq. (7) represents the equation of a straight line in a log-log plot of E_i/P_a against σ_3/P_a . The slope of the best fit line is n and the intercept at σ_3/P_a = 1.0 is K. Linear regression of these data are shown in Fig. 5; the straight lines shown correspond of to K=138 and n=0.79 for unconsolidated drained tests, and K= 159 and n=0.84 for consolidated drained tests.

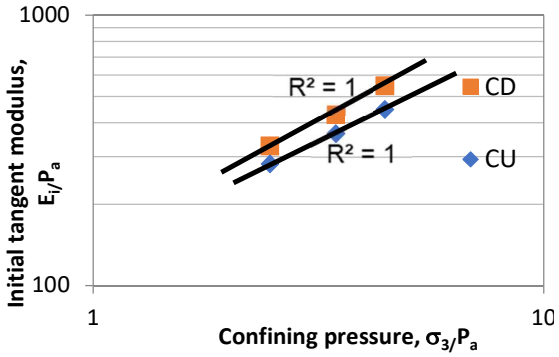


Fig. 5 Variation of initial tangent modulus with confining pressure for consolidated undrained and consolidated drained triaxial tests on Addis Ababa red clay soil (log-log plot).

Summary of the parameters: - The stress-strain parameters derived from the results of consolidated undrained and consolidated drained tests on the clay soil samples are summarized in Table 6. Theses parameters were used to back-predict the stress-strain behavior of the soil.

Table 6. Summary of hyperbolic stress-strain parameters for the investigated clay soil from CU and CD triaxial tests.

Type of triaxial test	Consolidated undrained	Consolidated drained
cohesion, (kPa)	20	15
Angle of internal friction, (deg)	19	21.5
Average failure ratio, R_f	0.875	0.968
Modulus number, K	138	159
modulus exponent, n	0.79	0.840

The hyperbolic stress-strain relationships are given by equations (1)-(5). Substituting equations (2), (4) and (5) into equation (1) yields:

$$(\sigma_1-\sigma_3) = \frac{\varepsilon}{\frac{1}{K P_a\left(\frac{\sigma_3}{P_a}\right)^n} + \frac{\varepsilon R_f(1-\sin\phi)}{2c\cos\phi + 2\sigma_3\sin\phi}} \qquad (8)$$

Using the determined hyperbolic parameters (K, n and R_f), the model's back-prediction of the stress- strain relationship was compared with the corresponding experimental results using Eq. (8). The comparison of the experimental and the calculated hyperbolic stress-strain curves for the consolidated undrained and consolidated drained tests are shown in Figs. (6) and (7) respectively.

The comparison showed that the hyperbolic model provides a very good accuracy to reproduce the stress-strain curves obtained from the test results of the two types of triaxial tests.

In general, the model shows strong capabilities to replicate the experimental results of the tested clay soil and capture the influence of confining pressure and drainage condition on the behavior of the soil.

For a given confining pressure and strain level, the hyperbolic parameters shown in Table 6 can be used to predict the deviatoric stress expected to develop in the soil under investigation when it is subjected to an additional load.

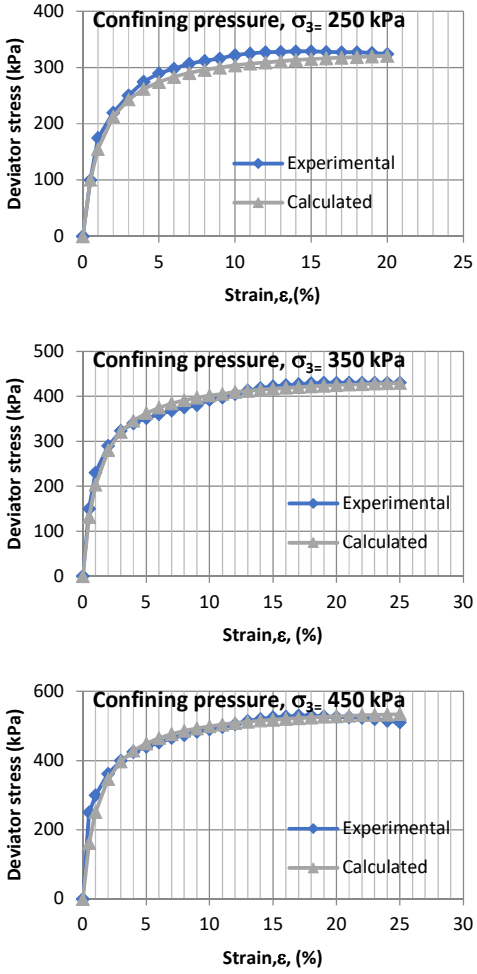


Fig. 6 Comparison of experimental and calculated stress-strain curves for consolidated undrained tests

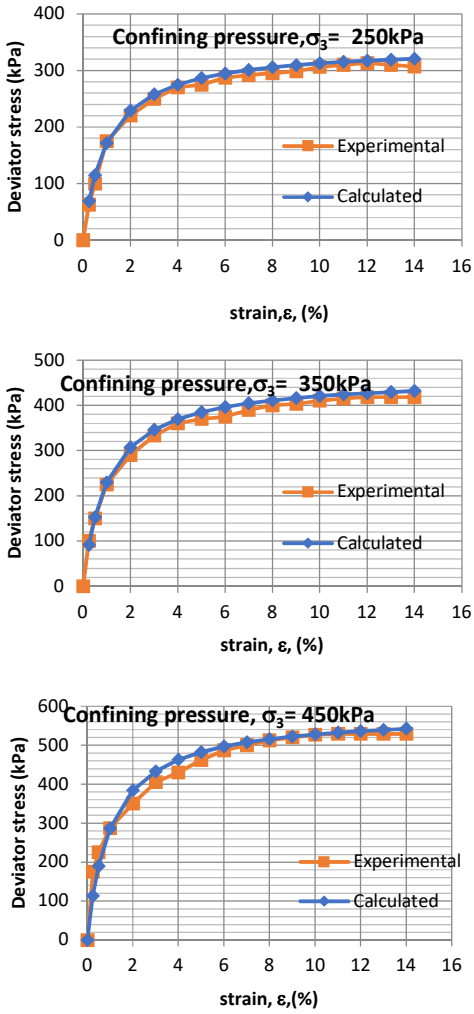


Fig. 7 Comparison of experimental and calculated stress-strain curves for consolidated drained tests

CONCLUSION

In this paper, a simple hyperbolic model of Duncan and Chang has been used to generate the complete deviator stress vs. vertical strain curves of the consolidated undrained and consolidated drained triaxial tests on clay soil samples collected from the North of Addis Ababa. The model has the advantage of considering the influence of confining pressure on the stress-strain curve characteristics.

The comparison of the calculated with the experimental test results show that the model is sufficient to reproduce with a very good accuracy the stress-strain behavior of the soil under investigation.

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