Comparative Study of Different Probabilistic Methods in the Analysis of Shallow Foundation Safety

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Abstract

With the aim of proposing better analysis methods for designing shallow foundations, this research compares different types of probabilistic analysis methods used to analyze foundation safety. Both deterministic and random variable input data have been used for this analysis which include the N values obtained from the SPT result (number of blows), the width of foundation, depth of foundation, and load of the building. The spatial interpolation (kriging) method estimates the soil data at unsampled locations to increase the quality of the data set for the probabilistic analysis. The variable N is treated as a random variable among the input variables, while the rest are treated as deterministic values. Foundation safety was evaluated for different foundation widths, and then compared with the target reliability index or probability of failure, and then, for each method, the proper width was chosen. The foundation width obtained from the probabilistic methods, the first-order reliability method, first-order second moment, and Monte Carlo simulation gives a similar result. In addition, the coefficient of variation has a significant effect on the value of the probability of failure. In general, the probabilistic method gives an excellent framework to incorporate the uncertainties of input variables.

Keywords: Kriging, FOSM, FORM, Monte Carlo Simulation

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Introduction

Engineering deals with real-world problems where uncertainties are inevitable. Coping with uncertainty is the hallmark of geotechnical engineering, considering the complex nature of soil and rock, changeable environmental conditions, and uncertainties in predicting field performance from available geotechnical models. Uncertainties that are dealt with in engineering can be classified into two broad types; those that are associated with natural randomness (aleatory) and those associated with our inaccuracy of prediction and estimation of reality (epistemic) (Hengle, 2009)

One of the practices in geotechnical engineering is evaluating the foundation safety for designing a shallow foundation. The accurate assessment of the foundation's reliability is essential for safe and reliable design, which also requires the precise estimation of soil properties.

Spatial interpolation

The purpose of spatial interpolation or spatial prediction is to predict values of the target variable throughout the entire area of interest. In statistics, originally in geostatistics, kriging or Kriging, also known as Gaussian process regression, is a method of interpolation based on Gaussian process governed by prior covariances. Under suitable assumptions of the prior, kriging gives the best linear unbiased prediction at unsampled locations. Kriging can be understood as a two-step process: first, the spatial covariance structure of the sampled points is determined by fitting a variogram; and second, weights derived from this covariance structure are used to interpolate values for unsampled points or blocks across the spatial field. Kriging has been used alternately been used with geostatistical interpolation, which is also called Best linear unbiased prediction. Kriging provides optimal linear prediction at non-observed locations by assuming that the unknown spatial variation of the property is a realization of a random function that has been observed at the data points only. Moreover, the predicted value kriging also provides variance of the prediction error. (Hengle, 2009)

Reliability analysis

First-order second moment method

The first-order second-moment method produces a linearization around the mean value of the input's random variables of a probabilistic problem. It uses the Taylor series expansion of the performance function to evaluate the first two central moments, mean and standard deviation values. The Taylor series expansion of the performance function is truncated after the first term assuming that the higher-order values beyond the first term are negligible. This linearizes the function; hence it's called first order. (H.-S. Ang & H. Tang, 1984) (Baecher& Christian, 2003)

If there are n uncertain variables, this method requires either evaluating n partial derivatives of the performance function or performing a numerical approximation using evaluations at 2n+1 points. Considering a performance function g with x_i number of random variables, the Taylor series expansion of the performance function truncated after the first term will be:

$$g(x1, x2..xn) \approx g(\mu x1, \mu x2 \dots \dots \mu xn) + \Sigma(xi - \mu x1)ni = 1 + \partial g \partial xi$$

After evaluating the derivative for each xi value, the approximate mean of the performance function is given by

$$\mu_g \approx g(\mu_{x1}, \mu_{x2} \dots \dots . \mu_{x3})$$

The variance of the performance function, gi

$$\sigma_g{}^2 \approx \sum \sigma_g{}^2 xi\left(\frac{\partial g}{\partial xi}^2\right) + \sum_{i=1}^{n}\sum_{j\neq 1}^{n} cov(Xi, Xj)\frac{\partial g}{\partial xi}\frac{\partial g}{\partial xj}$$

Then the mean and standard deviation of the performance function can be evaluated, which is used to compute the reliability index as $\beta = \mu g/\sigma g$ then the probability of failure is associated as $pf = \emptyset(-\beta)$.

First-order reliability analysis

The first-order reliability method (FORM), also known as The Hasofer–Lind Approach, is one of a method used to evaluate the reliability index and probability of failure. Hasofer–Lind proposes a geometric definition of the reliability index β as the shortest distance from the mean value point of the random variables to the limit state surface in units of directional standard deviations. (Hasofer & Lind, 1974)

$$\beta = \frac{min}{z=0}\sqrt{(U_R{}^2 + U_L{}^2)}$$

Evaluation of the reliability index of the Taylor expansion of the performance function Z, i.e., the linearization of the limit state equation, will be performed at a point that is called the most probable failure point. Then the mean and standard deviation of the performance function can be evaluated, which is used to compute the reliability index as $\beta = \mu g/\sigma g$ then the probability of failure is associated as $pf = \emptyset(-\beta)$.

Monte Carlo simulation

The required analytical model may become extremely difficult to formulate mathematically as the complexity of an engineering system increases unless gross idealization and simplifications are invoked; additionally, even if a formulation is possible, the required solution may be analytically intractable in some cases. Monte Carlo simulations can be used to generate a probabilistic solution in these cases. (Baecher & Christian, 2003)

Monte Carlo simulation is a straightforward approach that is compatible with numerical analysis and is appropriate for situations involving the evaluation of complex integrals that are too complicated to solve analytically. (Al-Bittar, 2012) (Kayser et. al., 2014)

The basic idea of this method is that random values of the input variables are sampled based on their probability distribution, and the number of samples falling into the failure domain (Nf) is identified ($g(X)$≤0), then failure probability *Pf*, is estimated from

$$pf = \frac{1}{N}\sum_{i=1}^{N} I(g(x))$$

Methodology

The area where a proposed condominium housing project is going to be built is selected for the analysis. It's located at Nifas silk-2 (Jemo-1) sub-city Addis Ababa. Four cases are proposed to evaluate the foundation's reliability by considering different load and soil characteristic value variations. Spatial interpolation (kriging) is performed to evaluate the soil data at the unsampled locations. Then the deterministic method is used to evaluate the foundation width satisfying the failure criteria. The probabilistic methods (Monte Carlo simulation, first-order reliability method, and first-order second-moment reliability) were used to evaluate the probability of failure and reliability index of the foundation satisfying the target value

of reliability index or probability of failure according to Eurocode.

The analysis is performed using STATGRAPHIC Centurion 19 for the spatial interpolation (kriging) evaluation. COMREL was used for the first-order reliability (FOSM) method. The Monte Carlo simulation and first-order second-moment methods are evaluated using a Microsoft Excel sheet. Finally, the foundation widths evaluated using different probabilistic methods and deterministic methods were compared, and conclusions were drawn.

There are 18 boreholes conducted on the site with a recorded blow count to a depth of a maximum of 30 m. The STATGRAPHIC software uses these boreholes' northing and easting coordinates and its measured blow count number (N) at each specific depth as input data to perform the kriging.

Spatial interpolation (kriging)

The first is selecting and arranging the borehole data according to their recorded depth. This input data is then plotted as a scatter data plot using the easting and northing coordinates of each borehole. Then the appropriate experimental variogram is selected from the most used models (Gaussian, exponential, spherical, power) to fit the data variance. In fitting the experimental variogram to the data, the maximum lag distance and the increment are adjusted to find the appropriate curve fit. The model with the highest R^2 value, which indicates the goodness of the curve fit to the data, is selected. Then the spatial interpolation (kriging) is done for the entire area for the fixed grid spacing.

The default grid spacing in the STATGRAPHIC software is 2.75m, but in this study, 5m by 5m grid spacing for the whole area is selected after it checked that the variance and coefficient of variation of the data between the predicted values using the default grid spacing and the predicted values using 5m by 5m grid spacing is relatively small.

Reliability analysis

The size of the investigated area proposed for the construction of 6 blocks is approximately 130 x 110 m or 1.43 ha where a single block constitutes 2B+G+15 building. The designers proposed the dimension of the building to be 54.4 m x 17.5 m. Even though the area is proposed for the construction of six blocks, the reliability analysis is performed for one block only. The same procedure can be followed to evaluate the foundation safety at other locations.

Table 1 Type of cases considered

Case	*Number of floors*	*Foundation Depth*	*Load (kN)*
1	*B+G+15*	*3.5 m*	*235,350*
2	*2B+G+15*	*6.5 m*	*249,194*
3	*3B+G+15*	*9.5 m*	*263,038*
4	*4B+G+15*	*12.5 m*	*276,882*

Even though the proposed building has two basements, in this project, the building with one basement, three basements, and four basements are additionally studied. When evaluating each case, the depth of the foundation is assumed to be deterministic.

Input variables

The input variables used for the reliability analysis are deterministic input variables and random variables. Depth of foundation, foundation width, and length are considered deterministic input values. While maintaining the depth of foundation fixed for all cases, parameter analysis is done for the width of foundation B value.

The SPT-N value is treated as a random variable with its probability distribution.

The appropriate probability distribution is selected after fitting the data to different probability distributions using a Probability plot and comparing their R^2value, which indicates the goodness of curve fit. The one with the highest R^2 value is selected. The N value data for each case considered above is fitted to Normal, Lognormal, Gumbel, and Uniform distributions and compared to their best of the curve fit value. Then the mean and standard deviation for the N value is obtained from the curve fit for the reliability analysis.

Analysis type

FOSM

In the First order reliability method (FOSM), to find the reliability index and probability of failure, the performance functions mean and the standard deviation is calculated for each case using the input variables. Then using the performance functions mean and standard deviation, the foundation's reliability index and probability of failure are calculated using a Microsoft excel sheet using $\beta = \mu g/\sigma g$ and $pf = \emptyset(-\beta)$. A parameter study of foundation width is conducted, varying the width from 10m to 20m. The reliability index and probability of failure obtained for each width calculated are compared with the target reliability index and probability of failure selected above.

FORM

To evaluate the first-order reliability method COMREL software is used. It's one of the STRUREL collection of programs for structural reliability analysis. The performance function used for the FORM analysis (designated as FLIM) is

$$FLIM(1) = IF(width > depth, (2 * (SPT/0.08) * (1 + 0.33 * (depth/width))), (2 * (SPT/0.08) * 1.33)) - (load/(width * length)$$

The N values mean and the standard deviation is used as input, and the other parameters, load, length, and depth, are considered deterministic. The parameter study is conducted to find the width of the foundation satisfying the target value. Then the probability of failure and reliability index is evaluated for each case, and each width varies.

MONTE CARLO

Monte Carlo simulation is conducted using a Microsoft Excel sheet. RAND () function on excel is used to randomly generate N value from its mean and standard deviation value. The foundation width is also randomly generated varying between 10 m up to 25 m. The rest of the variables are considered deterministic input variables. Then for every generated N value and foundation width, the performance function is evaluated. The number of samples generated for the MC simulation method is 10^6. Then the evaluated values are treated statistically to find the probability of failure and reliability index for every possible foundation width.

$P(failure/B) = (P(B/failure) * Pf)/P(B)$

$(B/failure) = n1nf$

$(B) = 1/nB$

$Pf = nfn$

n1 is the number of Monte Carlo simulation samples failure occurs for each specific foundation width B, n is the total number of Monte Carlo simulation samples, nf is the number of Monte Carlo simulation samples where failure occurs,

and nB is the number of possible foundation width. Then the reliability index and probability of failure are compared with the target reliability index and probability of failure to determine the foundation width that satisfies the target value.

recorded. Finally, 621 new borehole data points at the unsampled location at each depth up to 30 m are estimated. The sample calculation at 4.5m depth is shown in Figure 1 whereas the scattered data plot of boreholes after kriging is shown Figure 3.

Results

Spatial interpolation (kriging)

The spatial interpolation is conducted at every layer of which the SPT blow count is

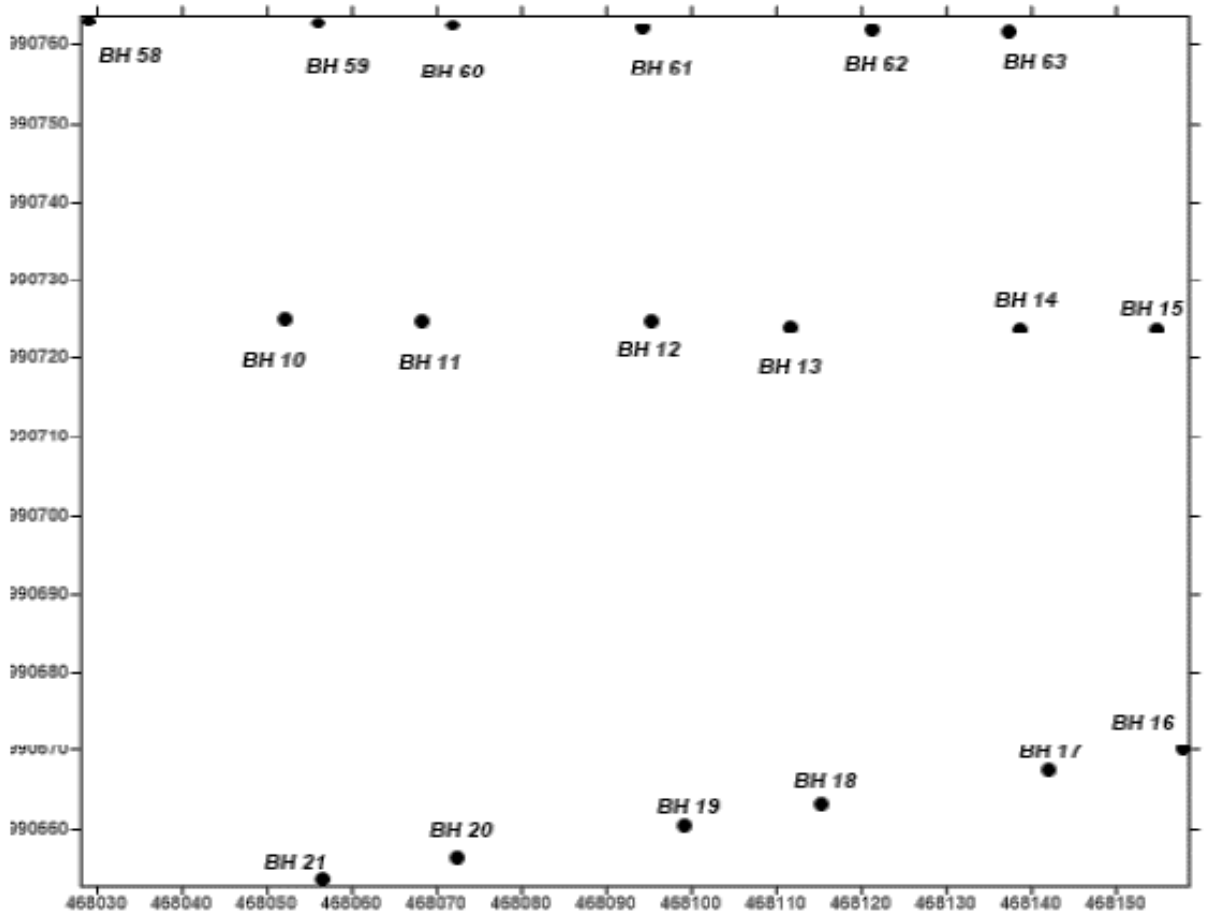


Figure 1 *Original borehole locations*

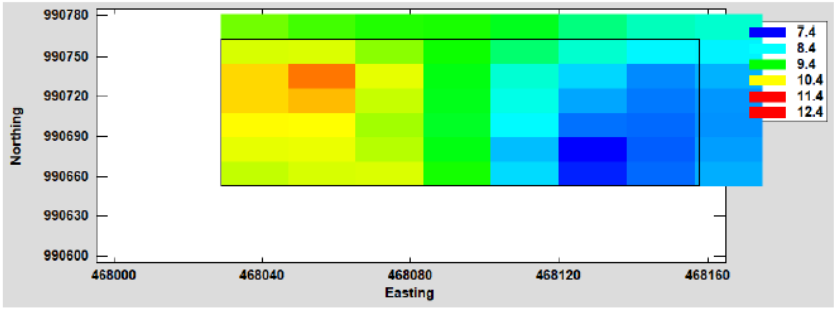


Figure 2 *Response map after kriging (at a depth of 4.5m)*

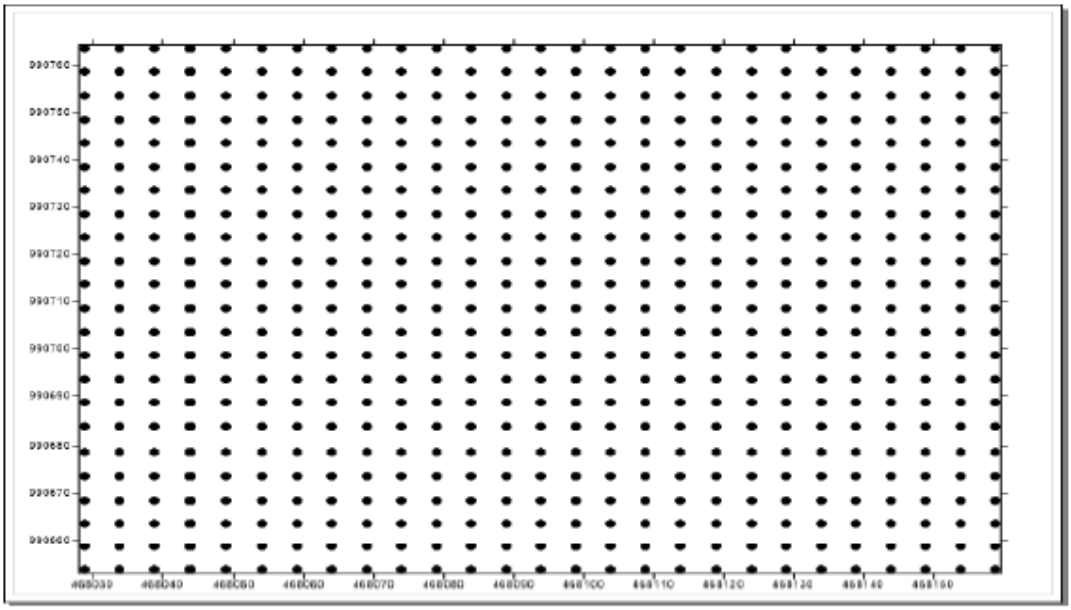


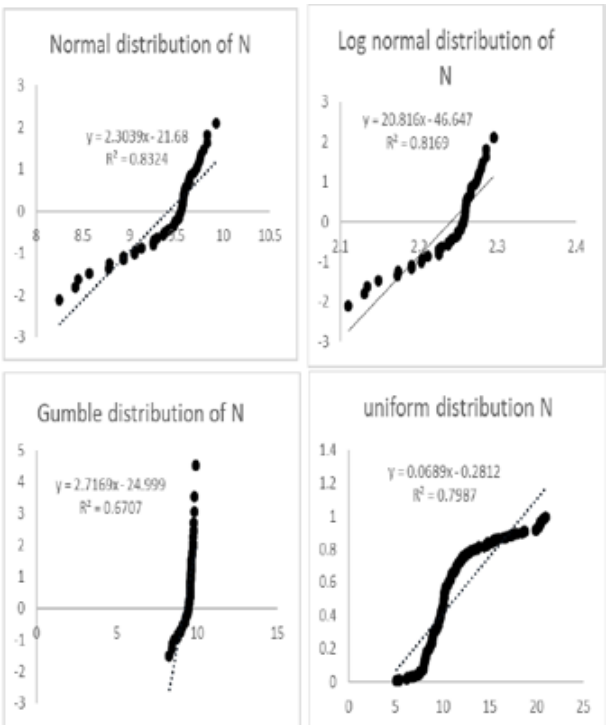
Figure 3 *Scattered data plot of boreholes after kriging*

Data characterization

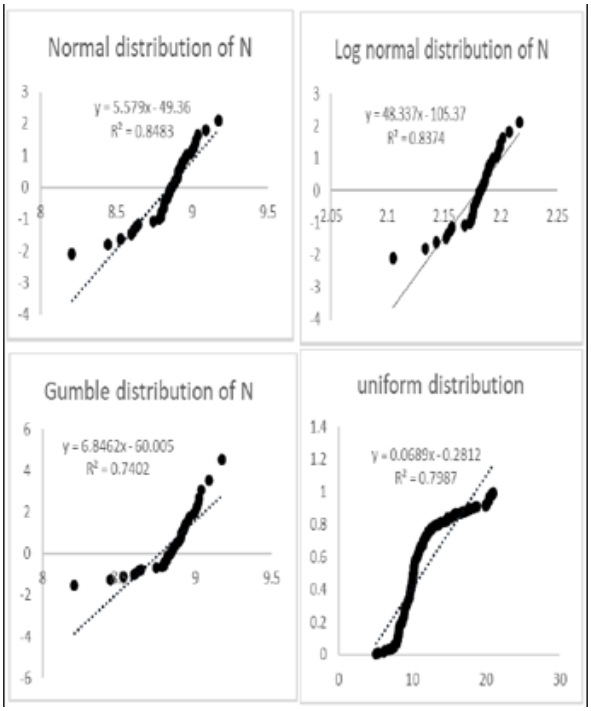
The kriging spatial interpolation produced more data points at unsampled locations. The N values under the foundation base area, and those close to the foundation are selected from the depth of the foundation up to the maximum of 30 m.

Considering the N values within the foundation influence zone and due to uncertainty in the standard penetration test, it is suggested that the method of the weighted average is proposed instead of using a particular N value. All the N value from the influence zone is taken under consideration by giving the highest weightage to the closest N value from the footing base. According to Bowles (1997), the influence depth is up to 2B, where B is the foundation width.

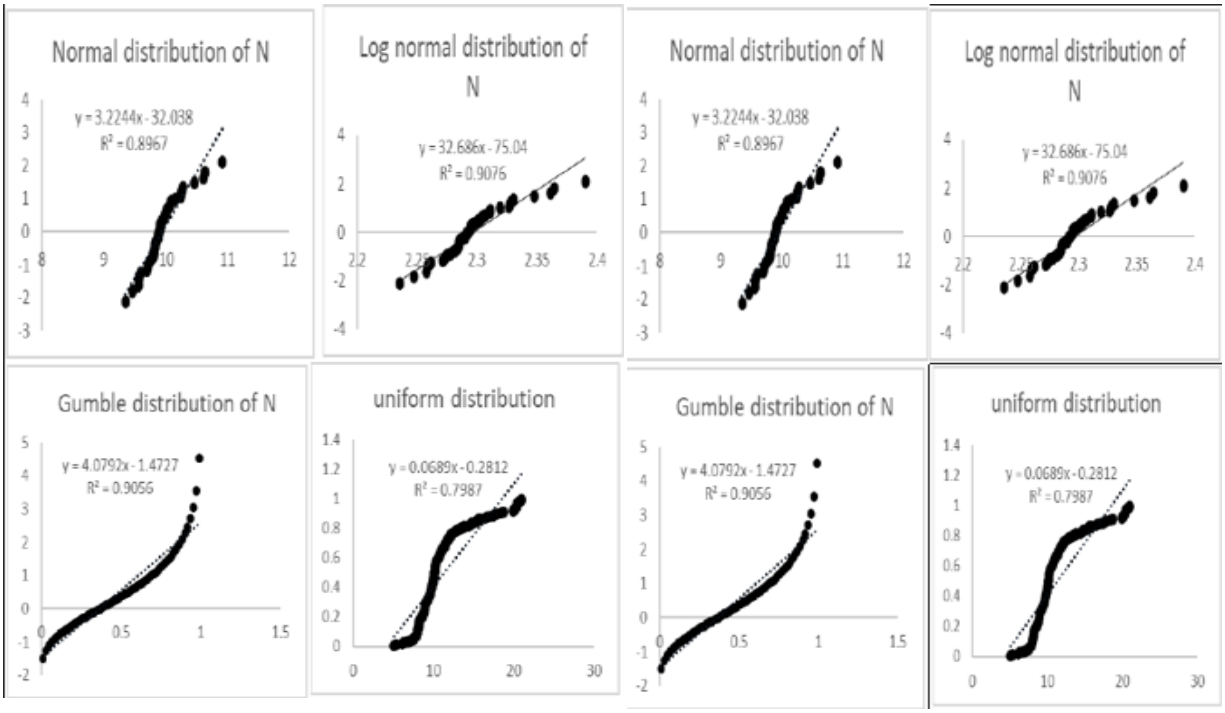
After the weighted average of the N, the value is evaluated for each case, and the data are fitted to different probability distribution curves to find the best curve fit from Normal, Lognormal, Gumbel, and Uniform probability distribution.



a) Data curve fit for case one



b) Data curve fit for case three



c) Data curve fit for case two

d) Data curve fit for case four

Figure 4 *Data Curve Fitting for all Cases*

The highest value of R^2 is selected for each case, which resulted in the selection of normal distribution for cases 1, 2, and 4, as well as lognormal distribution for case 3.

***Table 2** Input variables for case one*

Variable	Mean (μ)	St.Dev. (σ)	Prob. Distribution
N	9.4	0.43	Normal
B (m)	20 - 40	-	constant
D (m)	3.5m	-	constant
Load (kN)	235,350	-	constant

***Table 3** Input variables for case two*

Variables	Mean (μ)	St.Dev. (σ)	Prob. Distribution
N	8.8	0.18	Normal
B (m)	20 - 40	-	constant
D (m)	6.5	-	constant
Load (kN)	249,194	-	constant

***Table 4** Input variables for case three*

Variables	Mean (μ)	St.Dev (σ)	Prob. Distribution
N	9.9	0.31	Lognormal
B (m)	20 - 40	-	constant
D (m)	9.5	-	constant
Load (kN)	263,038	-	constant

***Table 5** Input variables for case four*

Variables	Mean (μ)	St.Dev. (σ)	Prob. Distribution
N	11	0.12	Normal
B (m)	20 – 40	-	constant
D (m)	12.5	-	constant
Load (kN)	401,480	-	constant

Reliability analysis

For the evaluation of the reliability analysis, the target reliability index selected is Eurocode Target reliability indices specified for the 50-year reference period (where 50 years equals the indicative design working life for common structure) for residential and office buildings is βT = 3.8, and the probability of failure Pf = 7.2 x 10^{-5} (ES EN 1990:2015)

After evaluating the reliability indices for each case, it's compared with the target reliability index and probability of failure.

First-order second-moment method (FOSM)

The result from the FOSM analysis is the reliability index and probability of failure for a different foundation width.

For cases one, two, three, and four at foundation widths of 21.5 m, 20.5 m, 19 m, and 17 m, the reliability index and probability of failure are greater than the target reliability index and probability of failure.

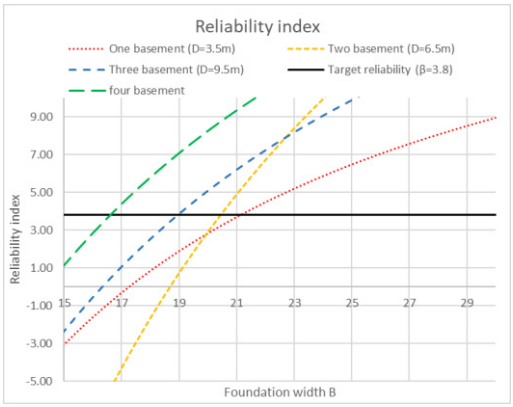


Figure 5 *Reliability index value from the FOSM method*

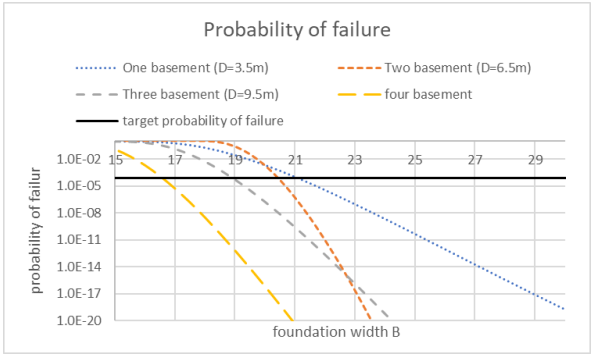


***Figure 6** Probability of failure value from the FOSM method*

First-order reliability method (FORM)

The software uses intuitive First Order Reliability approach and inbuilt solver with

constrained optimization to compute Reliability index and probability of failure.

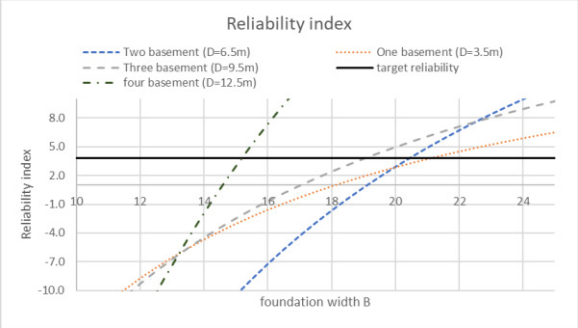


Figure 7 *Reliability of index value from FORM method*

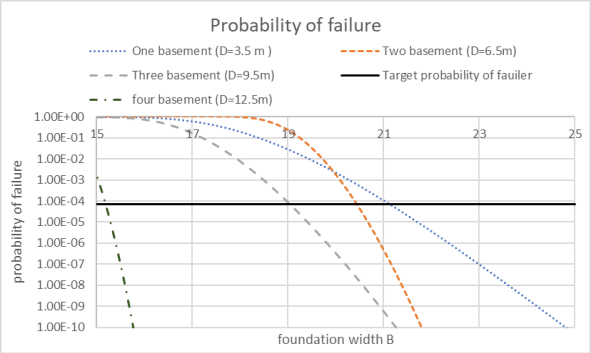


Figure 8 *Probability of failure value from FORM method*

For cases one, two, three, and four at foundation widths of 21.5 m, 20.5 m, 19.5m, and 15.5 m, the reliability index and probability of failure are greater than the target reliability index and probability of failure.

Monte Carlo simulation (MC)

In the Monte Carlo method, the number of samples that are required to generate for the simulation with the number of possible foundation widths of 101 is 14 million. But since the maximum number of samples allowed to generate in excel is 1 million, for low probability of failure values separate sheet is produced to generate a sample size of 500,000 for a single constant width. This solves the problem arising due to a small number of samples generated for a small value of the probability of failure.

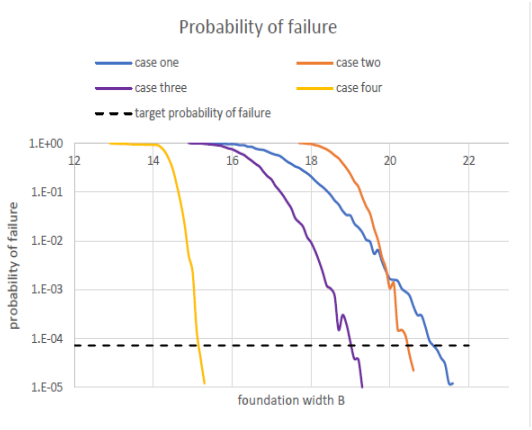


Figure 9 *Probability of failure value from Monte Carlo simulation*

For cases one, two, three, and four at foundation widths of 21.1 m, 20.5 m, 19.1 m, and 15.2 m, the probability of failure is greater than the target probability of failure.

Deterministic analysis

The deterministic analysis is done for two cases. One is for N cases without conducting spatial interpolation, while the other case is using the N value after conducting spatial interpolation.

Deterministic analysis without spatial interpolation

In this case, the N values are used obtained from the three boreholes found at the location of the proposed foundation. The weighted average of the N value below the foundation is calculated then, for each borehole, the N design is selected. Then using these values, the contact pressure is compared with the allowable bearing capacity obtained for the N design value. The N data obtained for the three boreholes is already a corrected value. Therefore, no correction is conducted.

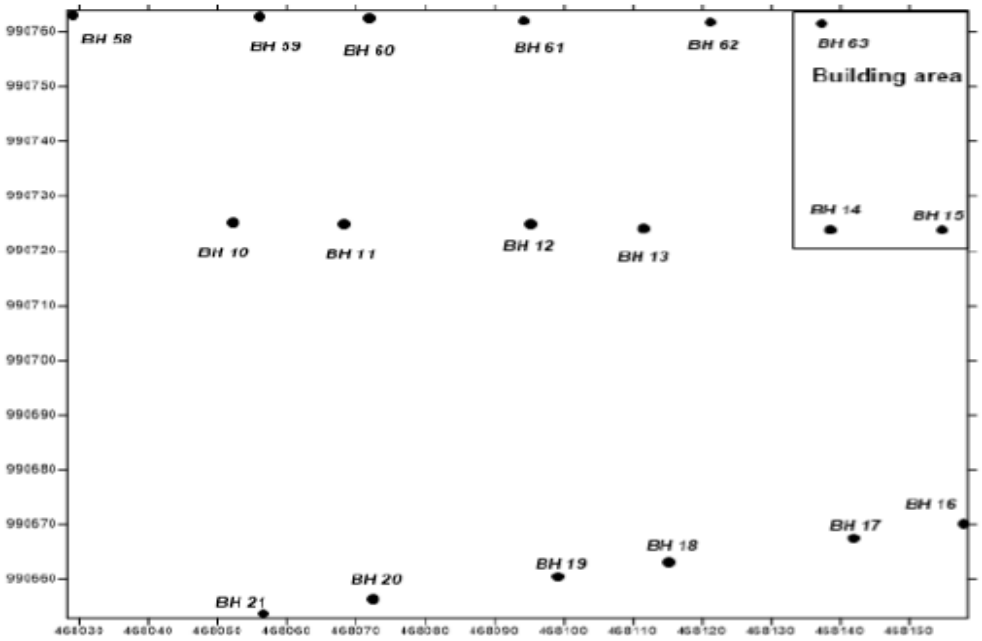


Figure 10 *Building area and borehole location*

Table 6 *SPT N data for BH 14, 15 and 63*

BH14	
Depth	N
4.5	8
6	12
7.5	8
9	9
10.5	10
12	14
13.5	10
15	12
16.5	11
18	13
19.5	12
21	10
22.5	20
24	20
25.5	4
27	3
28.5	5
30	12

BH15	
Depth	N
2	18
4	10
6	6
8	13
10	10
12	12
14	14
16	15
18	13
20	11
22	9
24	20
26	4
28	3
30	18

BH 63	
Depth	N
2	11
4	6
6	7
8	8
9.5	8
11	8
12.5	21
15	12
17	8
19	22
21	21
22.5	8
24	20
26	16
28	18

BH 14, BH 15, and BH 63 lie inside the foundation area, so from both boreholes, the N design is calculated for each case. After calculating the weighted average for each case, the minimum value from the three boreholes is selected as a design value for the foundation, as listed below:

- Case 1: N design = 7,
- Case 2: N design = 8
- Case 3: N design = 10
- Case 4: N design = 10

The foundation width for which Contact pressure is less than allowable bearing capacity is calculated.

Deterministic analysis with spatial interpolation

This analysis is done using the data after spatial interpolation (kriging) is conducted for the following load cases with corresponding N design and Loads:

- Case 1: N = 9.4, Load 235,350 kN
- Case 2: N = 8.8, Load 249,194 kN
- Case 3: N = 9.9, Load 263,038 kN
- Case 4: N = 11, Load 276,882 kN

The calculation is evaluated as above, i.e., calculating the foundation width in which the factor of safety is equal to one.

Summary of results

The results of the analysis are summarized below in Table 7. The results

show consistent changes as we move from Case 1 to Case 4 irrespective of the method utilized.

Table 7 *Foundation width (m) selected for each case*

Cases	1	2	3	4
FOSM	21.5	20.5	19	17
FORM	21.5	20.5	19.5	15.5
Monte Carlo	21.1	20.5	19.1	15.2
Deterministic before kriging	23.5	21	16.5	16.5
Deterministic after kriging	17.5	19	16.5	14.5

Discussion

The spatial interpolation is conducted using the ordinary kriging method, where it's assumed that the population's mean and standard deviation is constant but unknown. This method is a realistic assumption since the entire population's mean cannot be known unless the entire population is sampled; which is impossible to achieve in this physical environment.

The selection of the variogram to fit the data plays an important role in the estimated value of spatial interpolation. In this case, almost most of the variograms fitted to the data have R^2value above 95%.

The default value of the grid spacing for the STATGRAPHIC software is 2.75 m, but this selection is modified to grid space 5x5m to reduce the unnecessary data cluster when performing the analysis. And this selected spacing slightly affects the variance of the predicted value but compared to the default value; the selection made gives a relatively similar result as the default value. This is also because the N values within the area for each layer have similar values.

After kriging, the N value within the foundation area is selected for the reliability analysis. This approach reduces the coefficient of variation of the data values. But if the data value for the entire area were to be considered, the coefficient of variation percentage would be high. Since the foundation rests on a relatively small area considering the data value for the area where no foundation is going to be built is not accurate modeling.

Evaluating the weighted average to consider the N values below the foundation within the influence depth reduces the variability of the data depth-wise. This is because the N values found within the small foundation area have relatively low variation, and when the weighted average is evaluated, it reduces the variation within that depth.

The required foundation width to satisfy the target reliability obtained from the first-order reliability method and first-order second-moment methods are similar. This is because the performance function is linear. Thus, there is no need to approximate the Taylor series of the performance function for the FOSM method. This produces an exact solution to the performance function. So, when it's compared to the FORM method, which is suitable for linear and nonlinear performance functions, their result becomes similar.

For cases one and two, all probabilistic methods give similar results, whereas the deterministic analysis done after kriging gives a low estimate of foundation width compared with the deterministic analysis done before kriging. For case three, all probabilistic methods gave similar results and compared to deterministic results; they give a higher estimate. The first-order second moment result for case four is slightly higher than the first-order reliability method and Monte Carlo simulation.

In the Monte Carlo method, the number of samples that are required for the

simulation with several possible foundation widths of 101 is 14 million. But the maximum number of samples allowed to generate in excel is around 1 million. This is solved by using 1 million samples first, and then for low probability of failure values separate sheet is produced to generate a sample size of 500,000 for a single width.

The foundation width selected comparing its probability of failure or reliability index with the target value obtained from Eurocode satisfied not only the ultimate limit state criteria but also satisfies the serviceability limit state. This is due to Bowles formulas, which used the N value to obtain the allowable bearing capacity considering a settlement ratio that is set to a value that is allowed in the building code. Since 50 mm settlement is recommended for the mat foundation in the euro code, the formula implicitly satisfies the serviceability limit state.

Conclusions

Estimating the N values at the unsampled location using spatial interpolation (kriging) improves the probability method by producing more data for the analysis.

Since the SPT N data has low variation, a small increment in the N value significantly affects the probability of failure and the reliability index of the foundation. Therefore, great care must be given when evaluating the standard deviation (or the coefficient of variations) of the input variable.

FOSM and FORM methods give the same result because the performance function is linear, and no approximation is needed for the Taylor series of the performance function.

Monte Carlo simulation result is similar to FOSM and FORM probability methods for all cases because the performance function is linear and only has one random variable.

There is a need to develop better solutions due to Monte Carlo's main drawback, which is its high computing cost. As FORM often only requires a few iterations to reach convergence, it is computationally more effective than MCS. This is especially true when there is little chance of failure. With the current state of the research, it is safe to suggest that FORM might be used as a starting point in reliability-based design to examine the relative significance of parameters.

References

Al-Bittar, T. (2012). Probabilistic Analysis of Shallow Foundations Resting on Spatially Varying Soils.

Ang, H.-S., & Tang, H. (1984). P*robability Concepts in Engineering Planning and Design VOLUME II DECISION, RISK, AND RELIABILITY.*

Baecher, G. B., & Christian, J. T. (2003). *Reliability and Statistics in Geotechnical Engineering.*

Bowles, J. (1997). *Foundation Analysis and Design.*

ES EN 1990:2015. (2015). *2015. ES EN 0, Basis of Structural Design.*

Hasofer, A., & Lind, N. (1974). Exact and Invariant First Order Reliability Format.

Hengl, T. (2009). *Practical Guide to Geostatistical Mapping.*

Kayser, Mohammad, & Gajan. (2014). *Application of Probabilistic Methods to Characterize Soil Variability and Their Effects on Bearing Capacity and Settlement of Shallow Foundations: State of the Art.*